Chapter 10: Equations

Equation 10.1:

$$Y_i = \alpha + \beta C_i + \varepsilon_i$$

Equation 10.2:

$$C_i = \kappa + \lambda Y_i$$

Equation 10.3:

$$C_i = \frac{1}{1-\lambda\beta}\left(\kappa + \lambda\alpha + \lambda\varepsilon_i\right)$$

Equation 10.4:

$$y_i = \alpha + \beta x_i^* + \varepsilon_i$$

Equation 10.5:

$$x_i = x_i^* + \nu_i$$

Equation 10.6:

$$x_i^* = x_i - \nu_i$$

Equation 10.7:

$$y_i = \alpha + \beta\left(x_i - \nu_i\right) + \varepsilon_i = \alpha + \beta x_i + \left(\varepsilon_i - \beta\nu_i\right)$$

Equation 10.8:

$$y_i = \alpha + \beta a_i + \varepsilon_i$$

Equation 10.9:

$$x_i = \gamma + \delta a_i + \nu_i$$

Equation 10.10:

$$a_1 = \frac{1}{\delta}\left(x_i - \gamma - \nu_i\right)$$

Equation 10.11:

$$y_i = \left(\alpha + \frac{\beta\gamma}{\delta}\right) + \frac{\beta}{\delta}x_i + \left(\varepsilon_i - \frac{\beta}{\delta}\nu_i\right)$$

Equation 10.12:

$$\mathrm{COV}\left(x_i, \varepsilon_i\right) \neq 0$$

Equation 10.13:

$$\mathrm{E}\left(b\right) = \beta + \mathrm{E}\left(\frac{\sum_{i=1}^{n}\left(x_i - \overline{x}\right)\varepsilon_i}{\sum_{i=1}^{n}\left(x_i - \overline{x}\right)x_i}\right)$$

Equation 10.14:

$$\mathrm{E}(b) = \beta + \frac{1}{\sum_{i=1}^{n}(x_i - \bar{x})x_i} \sum_{i=1}^{n} \mathrm{E}\left((x_i - \bar{x})\varepsilon_i\right)$$

Equation 10.15:

$$\mathrm{E}\left((x_i - \bar{x})\varepsilon_i\right) = \mathrm{E}\left((x_i - \bar{x})(\varepsilon_i - 0)\right) = \mathrm{E}\left((x_i - \bar{x})\left(\varepsilon_i - \mathrm{E}(\varepsilon_i)\right)\right)$$

Equation 10.16:

$$\mathrm{E}\left[\left(x_i - \mathrm{E}(x_i)\right)\left(\varepsilon_i - \mathrm{E}(\varepsilon_i)\right)\right] = \mathrm{COV}(x_i, \varepsilon_i)$$

Equation 10.17:

$$y_i = -14,312 + 3,564.5x_i + e_i$$

Equation 10.18:

$$b \to \beta \frac{\mathrm{V}(x_i^*)}{\mathrm{V}(x_i^*) + \mathrm{V}(\nu_i)} < \beta$$

Equation 10.19:

$$\mathrm{COV}(x_i, z_i) \neq 0$$

Equation 10.20:

$$\mathrm{COV}(\varepsilon_i, z_i) = 0$$

Equation 10.21:

$$x_i = c + dz_i + f_i$$

Equation 10.22:

$$\hat{x}_i = c + dz_i$$

Equation 10.23:

$$y_i = a_{2\text{SLS}} + b_{2\text{SLS}}\hat{x}_i + e_i$$

Equation 10.24:

$$b_{2\text{SLS}} = \frac{\sum_{i=1}^{n}\left(\hat{x}_i - \overline{\hat{x}}\right)y_i}{\sum_{i=1}^{n}\left(\hat{x}_i - \overline{\hat{x}}\right)\hat{x}_i}$$

Equation 10.25:

$$a_{2\text{SLS}} = \overline{y} - b_{2\text{SLS}}\overline{\hat{x}}$$

Equation 10.26:

$$y_i = -19,802 + 3,986.8x_i + e_i$$

Equation 10.27:

$$\bar{\hat{x}} = \frac{\sum_{i=1}^{n}\left(c + dz_i\right)}{n} = c + d\,\frac{\sum_{i=1}^{n} z_i}{n} = c + d\bar{z}$$

Equation 10.28:

$$b_{2\text{SLS}} = \frac{d\sum_{i=1}^{n}\left(z_i - \bar{z}\right)y_i}{d\sum_{i=1}^{n}\left(z_i - \bar{z}\right)\hat{x}_i} = \frac{\sum_{i=1}^{n}\left(z_i - \bar{z}\right)y_i}{\sum_{i=1}^{n}\left(z_i - \bar{z}\right)\hat{x}_i}$$

Equation 10.29:

$$\sum_{i=1}^{n}\left(z_i - \bar{z}\right)f_i = 0$$

Equation 10.30:

$$b_{2\text{SLS}} = \frac{\sum_{i=1}^{n}\left(z_i - \bar{z}\right)y_i}{\sum_{i=1}^{n}\left(z_i - \bar{z}\right)x_i}$$

Equation 10.31:

$$a_{2\text{SLS}} = \bar{y} - b_{2\text{SLS}}\bar{x}$$

Equation 10.32:

$$b_{\text{IV}} = b_{2\text{SLS}}$$

Equation 10.33:

$$b_{\mathrm{IV}} = \frac{\sum_{i=1}^{n}\left(z_i - \overline{z}\right) y_i}{\sum_{i=1}^{n}\left(z_i - \overline{z}\right) x_i} = \frac{\sum_{i=1}^{n}\left(z_i - \overline{z}\right)\left(\alpha + \beta x_i + \varepsilon_i\right)}{\sum_{i=1}^{n}\left(z_i - \overline{z}\right) x_i}$$

Equation 10.34:

$$b_{\mathrm{IV}} = \beta + \frac{\sum_{i=1}^{n}\left(z_i - \overline{z}\right)\varepsilon_i}{\sum_{i=1}^{n}\left(z_i - \overline{z}\right) x_i}$$

Equation 10.35:

$$\mathrm{E}\left(b_{\mathrm{IV}}\right) = \beta + \mathrm{E}\left(\frac{\sum_{i=1}^{n}\left(z_i - \overline{z}\right)\varepsilon_i}{\sum_{i=1}^{n}\left(z_i - \overline{z}\right) x_i}\right)$$

Equation 10.36:

$$\mathrm{E}\left(b_{\mathrm{IV}}\right) = \beta + \frac{1}{\sum_{i=1}^{n}\left(z_i - \overline{z}\right) x_i} \sum_{i=1}^{n} \mathrm{E}\left(\left(z_i - \overline{z}\right)\varepsilon_i\right)$$

Equation 10.37:

$$\mathrm{V}\left(b_{\mathrm{IV}}\right) = \mathrm{V}\left(\frac{\sum_{i=1}^{n}\left(z_i - \overline{z}\right) y_i}{\sum_{i=1}^{n}\left(z_i - \overline{z}\right) x_i}\right)$$

Equation 10.38:

$$\mathrm{V}\left(b_{\mathrm{IV}}\right) \approx \frac{1}{\left(\sum_{i=1}^{n}\left(z_i-\overline{z}\right)x_i\right)^2}\mathrm{V}\left(\sum_{i=1}^{n}\left(z_i-\overline{z}\right)y_i\right)$$

Equation 10.39:

$$\mathrm{V}\left(b_{\mathrm{IV}}\right) \approx \frac{\sigma^2\sum_{i=1}^{n}\left(z_i-\overline{z}\right)^2}{\left(\sum_{i=1}^{n}\left(z_i-\overline{z}\right)x_i\right)^2}$$

Equation 10.40:

$$s_{\mathrm{IV}}^2 = \frac{\sum_{i=1}^{n}e_i^2}{n-2} = \frac{\sum_{i=1}^{n}\left(y_i-a_{\mathrm{IV}}-b_{\mathrm{IV}}x_i\right)^2}{n-2}$$

Equation 10.41:

$$\mathrm{V}\left(b_{\mathrm{IV}}\right) = \frac{s_{\mathrm{IV}}^2\sum_{i=1}^{n}\left(z_i-\overline{z}\right)^2}{\left(\sum_{i=1}^{n}\left(z_i-\overline{z}\right)x_i\right)^2}$$

Equation 10.42:

$$\mathrm{SD}\left(b_{\mathrm{IV}}\right) = +\sqrt{\frac{s_{\mathrm{IV}}^2\sum_{i=1}^{n}\left(z_i-\overline{z}\right)^2}{\left(\sum_{i=1}^{n}\left(z_i-\overline{z}\right)x_i\right)^2}}$$

Equation 10.43:

$$\mathrm{V}\left(b_{\mathrm{IV}}\right)=\frac{\sigma^2}{n\ \mathrm{V}\left(x_i\right)\left(\mathrm{CORR}\left(z_i,x_i\right)\right)^2}$$

Equation 10.44:

$$\mathrm{V}\left(b\right)=\frac{\sigma^2}{\sum_{i=1}^{n}\left(x_i-\bar{x}\right)^2}\approx\frac{\sigma^2}{n\ \mathrm{V}\left(x_i\right)}$$

Equation 10.45:

$$\mathrm{V}\left(b_{\mathrm{IV}}\right)=\frac{\mathrm{V}\left(b\right)}{\left(\mathrm{CORR}\left(z_i,x_i\right)\right)^2}\geq\mathrm{V}\left(b\right)$$

Equation 10.46:

$$\frac{\mathrm{CORR}\left(z_i,\varepsilon_i\right)}{\mathrm{CORR}\left(z_i,x_i\right)}$$

Equation 10.47:

$$y_i=a+b_1x_i+b_2\hat{x}_i+e_i.$$

Equation 10.48:

$$\text{years of schooling}=12.542-1.3085\left(\text{allocation code}\right)+e_i$$
$$\left(1,329\right)\quad\left(39.89\right)$$

Equation 10.49:

$$\underset{\quad}{\text{earnings}} = \underset{(17.89)}{-67{,}168} + \underset{(26.07)}{7{,}869.5}(\text{predicted years of schooling}) + e_i$$